MATH-312 Mathematical Methods

Credit Hours: 3-0

Prerequisites: MATH-251 Ordinary Differential Equations

Course Objectives: The course provides an in depth knowledge of some topics in applied mathematics that permeate various scientific disciplines. It includes techniques to derive power series solutions of ODEs with variable coefficients. Laplace transform and its application in treating of ODEs are also covered. Fourier Series, Sturm-Liouville problems and Variational problems are also included as core contents.

Core Contents: Series Solutions, Laplace Transform, Fourier Series, SturmLiouville Problems, Variational problems.

Detailed Course Contents: Series Solutions: Review of Power Series, Ordinary and Singular Points, Power-Series Solutions about Ordinary and Singular Points, Bessel Function and its Properties, Legendre's Equation. Laplace Transform: Definition of Laplace Transform, Transform of Derivatives, Properties of Laplace Transform, Systems of Linear ODEs. Green's Function: Boundary Value Problems Involving Ordinary Differential Equations, Construction of Green's Function, Green's Function for Ordinary Differential Equations. Fourier Series, Sturm-Liouville Problems, Variational problems.

Course Outcomes: Upon completion of this course, the student should be able to:

- Understand the power series method and implement it for solving ordinary differential equations.
- Construct Green's function and use it to solve a variety of ODEs Understand basics of calculus of variations.

Text Books:

1. Dennis G. Zill and Michael R. Cullen: Differential equations with boundary value problems, (7th Edition), 2009, Brooks/Cole Cengage Learning, Canada (Referred as Zill)
2. A. J. Jerri, Introduction to integral equations with applications, John Wiley & Sons,1999 (Referred as AJ)
3. Bruce van Brunt, The Calculus of Variations, Springer-verlag New York Inc, 2004 (Referred as BB)

Reference Books:

1. N. Finizio and G. Ladas, An Introduction to Differential Equations, Wadsworth Publishing Company Belmont, California, 1982
2. William E. Boyce and Richard C. DiPrima: Elementary Differential Equations

and Boundary Value Problems (9th Edition), 2009, John Wiley & Sons, Inc.
3. K.T. Tang, Mathematical Methods for Engineers and Scientists Vol 3, Springer-Verlag Berlin Heidelberg 2007
4. Louis Komzsik, Applied Calculus of Variations for Engineers, CRC Press, 2009.
5. Bernard Dacorogna, Introduction to the calculus of variations, World Scientific Publishing, 1992.
6. Robert Weinstock, Calculus of variations with applications to physics and engineering, McGraw-Hill, 1952

Weekly Breakdown		
Week	**Section**	**Topics**
1	6.1.1, 6.1.2 (Zill)	Review of Power-Series; Ordinary and Singular Points; Power-Series Solutions about Ordinary Points
2	6.2	Regular and Irregular Singular Points; Classification of Singular Points; Frobenius Theorem
3	6.3.1	Bessel's Equation; Bessel functions of first and second kinds; Properties of Bessel functions and their Applications
4	6.3.2	Legendre's Equation; Legendre's Polynomials and their Properties
5	7.1, 7.2.1, 7.2.2	Definition of the Laplace Transform; Inverse Transform; Transform of Derivatives
6	7.3.1,7.3.2	Translation of the t−axis; Translation on the x−axis; Solving Initial Value Problems using Laplace Transform
7	7.4.1-7.4.3, 7.5	Derivative of a Transform; Convolution Theorem; Transform of Periodic Functions, The Dirac Delta Function, Initial-Value Problems, Systems of Linear Differential Equations
8	4.1(AJJ)	Construction of the Green's Functions: Non-homogeneous differential equations, Construction by variation of parameter, orthogonal representation,
9	**Mid Semester Exam**	
10	4.2	Boundary value problems and Greem's function.
11	11.1,11.2 (Zill)	Orthogonal Functions, Fourier Series
12	11.3, 11.4	Fourier Cosine and Sine Serie, Sturm-Liouville problems
13	11.5	Bessel and Legendre Series: Fourier-Bessel Series, FourierLegendre Series
14	2.1, 2.2	The First Variation: The Finite-Dimensional Case, The EulerLagrange Equation
15	2.3-2.5	Some Special Cases, A Degenerate Case, Invariance of the Euler-Lagrange Equation
16	3.1-3.3	Some Generalizations: Functionals Containing Higher-Order Derivatives, Several Dependent Variables Two Independent Variables
17	4.1, 4.2	Isoperimetric Problems: The Finite-Dimensional Case and Lagrange Multipliers, The Isoperimetric Problem.
18	**End Semester Exam**	