MATH-956 Finite Volume Method

Credit Hours: 3-0

Prerequisite: None

Course Objectives: This course aims on a powerful class of numerical methods for approximating solution of hyperbolic partial differential equations, including both linear problems and nonlinear conservation laws.

Core Contents: Conservation laws, Finite volume methods, Multidimensional problems. Linear waves and discontinuous media. The advection equation. Diffusion and the advection– diffusion equation, Nonlinear equations in fluid dynamics. Linear acoustics, Sound waves. Hyperbolicity of linear systems, Variable-coefficient hyperbolic systems. Solution to the Cauchy problem. Superposition of waves and characteristic variables, Left eigenvectors, Simple waves, Acoustics, Domain of dependence and range of influence. Discontinuoussolutions, The Riemann problem for a linear system. Coupled acoustics and advection,Initial–boundary-value problems.

General formulation for conservation laws, A numerical flux for the diffusion equation, Necessary components for convergence, The CFL condition. An unstable flux, The Lax– Friedrichs method, The Richtmyer two-step Lax–Wendroff method, Upwind methods, The upwind method for advection. Godunov's method for linear systems, The numerical flux function for Godunov's method. Flux- difference vs. flux-vector splitting, Roe's method. The Lax–Wendroff method, The beam–warming method, Preview of limiters. Choice of slopes, Oscillations, Total variation. Slope-limiter methods, Flux formulation with piecewise linear reconstruction, Flux limiters, TVD limiters

Course Outcomes: Students are expected to understand the various variants of the of finite volume method and its applications to problems like:

- Linear waves and discontinuous media.
- Diffusion and the advection–diffusion equation.
- Coupled acoustics and advection.

Text Book: Randall J. Leveque, Finite Volume Methods for Hyperbolic, Problems, Cambridge University Press, (2004)

Reference Books: F. Moukalled, L. Mangani, M. Darwish, “The Finite Volume Method in Computational Fluid Dynamics’’, Springer, 2016

ASSESSMENT SYSTEM

Nature of assessment	Frequency	Weightage (%age)
Quizzes	Minimum 3	10-15
Assignments	-	5-10
Midterm	1	25-35
End Semester Examination	1	40-50
Project(s)	-	10-20

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.3	Conservation laws, Finite volume methods, Multidimensional problems.
2	1.4.2.1	Linear waves and discontinuous media. The advection equation.
3	2.2,2.6	Diffusion and the advection–diffusion equation, Nonlinear equations influid dynamics.
4	2.7,2.8	Linear acoustics, Sound waves.
5	2.9,2.10,3.1	Hyperbolicity of linear systems, Variable-coefficient hyperbolicsystems. Solution to the Cauchy problem.
6	3.2-3.6	Superposition of waves and characteristic variables, Left eigenvectors, Simplewaves, Acoustics, Domain of dependence and range of influence.
7	3.7,3.8	Discontinuous solutions, The Riemann problemfor a linear system
8	3.10,3.11	Coupled acoustics and advection, Initial–boundary-value problems.
9	**Mid Semester Exam**	
10	4.1-4.4	General formulation for conservation laws, A numerical flux for the diffusionequation, Necessary components for convergence, The CFL condition.
11	4.5-4.9	An unstable flux, The Lax–Friedrichs method, The Richtmyer two-step Lax–Wendroff method, Upwind methods, The upwind

		method for advection.
12	4.10,4.11	Godunov's method for linear systems, The numerical flux function forGodunov's method.
13	4.13, 4.14	Flux-difference vs. flux-vector splitting, Roe's method
14	6.1-6.3	The Lax–Wendroff method, The beam–warming method, Preview oflimiters.
15	6.5-6.7	Choice of slopes, Oscillations, Total variation.
16	6.9-6.12	Slope-limiter methods, Flux formulation with piecewise linearreconstruction, Flux limiters, TVD limiters
17	-	Review
18	**End Semester Exam**	