MATH-455 Integral Equations

Credit Hours: 3-0
Prerequisites: None

Course Objectives: Integral equations have been of considerable significance in the history of mathematics. This course is mainly concerned with linear integral equations and a brief discussion of a simple type of non-linear the connection between differential and integral equations.

Core Contents: Classification of integral equations, Connection with differential equations, Integral equations of the convolution type, Method of successive approximations, Integral equations with singular kernels, Fredholm theory, Hibert-Schmidt theory

Detailed Course Contents: Classification of integral equations: Linear integral equations, Special types of kernel, Square integrable functions and kernels, Singular integral equations, Non-linear equations.
Connection with differential equations: Linear differential, Green's function, Influence function.
Integral equations of the convolution type: Integral transforms, Fredholm equation of the second kind, Volterra equation of the second kind, Fredholm equation of the first kind, Volterra equation of the first.
Method of successive approximations: Neumann series, Iterates and the resolvent kernel
Integral equations with singular kernels: Generalization to higher dimensions, Green's functions in two and three dimensions, Dirichlet's problem, Poisson's formula for the unit disc, Poisson's formula for the half plane, Hilbert kernel, Singular integral equation of Hilbert type.
Thee resolvent: Resolvent equation, Uniqueness theorem, Characteristic values and functions, Neumann series, Volterra integral equation of the second kind, Bacher's example, Fredholm equation in abstract Hilbert space.
Fredholm theory: Degenerate kernels, Approximation by degenerate kernels, Fredholm theorems, Fredholm theorems for completely continuous operators.
Fredholm formulae for continuous kernels.
Hibert-Schmidt theory, Hermitian kernels, Spectrum of a Hilbert-Schmidt kernel, Expansion theorems, Hilbert-Schmidt theorem, Solution of Fredholm equation of second kind, Bounds on characteristic values, Positive kernels, Mercer's theorem, Variational principles, Rayleigh-Ritz variational method.

Course Outcomes: The students will be able to

- Understand the theory of linear integral equations
- Apply different techniques to solve integral equations
- Understand the connection between differential and integral equations

Text Book: B. L. Moiseiwitsch, Integral equations, Longman London and New York, 1977.

Reference Books:

1. R. P. Kanwal, Linear Integral Equations Theory and Technique, Academic Press 1971.
2. Abdul-Majid Wazwaz, Linear and Nonlinear Integral Equations: Methods and Applications, Springer 2011.
3. Hochstadt, Integral equations, John Wiley and Sons, 1973.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.3	Classification of integral equations, Historical introduction, Linear integral equations , Special types of kernel , Symmetric kernels, Kernels producing convolution integrals, Separable kernels.
2	1.4-1.6	Square integrable functions and kernels ,Singular integral equations Non-linear equations.
3	2.1-2.3	Linear differential equations, Green's function, Influence function.
4	3.1-3.3	Integral transforms, Fredholm equation of the second kind, Volterra equation of the second kind.
5	3.4-3.6	Fredholm equation of the first kind, Stieltjes integral equation, Volterra equation of the first kind.
6	**First One Hour Test**	
7	4.1-4.2	Method of successive approximations: Neumann series, Iterates and the resolvent kernel.
8	5.1-5.2	Generalization to higher dimensions, Green's functions in two and three dimensions.
9	5.3, 5.4	Dirichlet's problem, Poisson's formula for the unit disc , Poisson's formula for the half plane, Hilbert kernel, Hilbert transforms, Singular integral equation of Hilbert type.
10	8.1-8.3	Resolvent equation, Uniqueness theorem, Characteristic values and functions
11	8.4,8.5	Neumann series, Volterra integral equation of the second kind, Bacher's example, Fredholm equation in abstract Hilbert space.
12	**Second One Hour Test**	
13	9.1, 9.2	Degenerate kernels, Approximation by degenerate kernels.
14	9.3, 9.4	Fredholm, theorems, Fredholm theorems for completely continuous, Operators, Fredholm formulae for continuous kernels.
15	10.1, 0.2	Hermitian kernels, Spectrum of a Hilbert-Schmidt kernel.
16	10.3, 10.4	Expansion theorems, Hilbert-Schmidt theorem, Hilbert's formula, Expansion theorem for iterated kernels, Solution of Fredholm equation of second kind.
17	10.5-10.8	Bounds on characteristic values, Positive kernels, Mercer's theorem, Variational principles, Rayleigh-Ritz variational method.
18	**End Semester Exam**	